INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - Third Year, 2012-13
Statistics - IV, Midterm Examination, March 4, 2013
Marks are shown in square brackets.   Total Marks: 50

1. Consider an $I \times J$ contingency table where the (i, j) cell has probability p_{ij}. Find the maximum likelihood estimate of p_{ij}
(a) when no restrictions are placed on the row and column factors;
(b) when it is known that the row and column factors are independent. [10]

2. (a) Suppose X_1 and X_2 are i.i.d. continuous random variables and U is a continuous positive random variable independent of X_1. Let $X_3 = X_1 + U$. Show that X_3 is stochastically larger than X_2.
(b) Suppose $X \sim \text{Gamma}(\lambda, \alpha)$ and $Y \sim \text{Gamma}(\lambda, \delta)$ with density,
$f_X(x) = \frac{\lambda^\alpha}{\Gamma(\alpha)} \exp(-\lambda x) x^{\alpha-1}$, $f_Y(x) = \frac{\lambda^\delta}{\Gamma(\delta)} \exp(-\lambda x) x^{\delta-1}$, for $x > 0$, where $\lambda > 0$, $\alpha > 0$, $\delta > 0$.
Show that Y is stochastically larger than X if $\delta > \alpha$.   [10]

3. Let $U_{(i)}^{(n)}$ denote the ith order statistics from a random sample of size n from $U(0, 1)$. Show that, for each i, $1 \leq i \leq n$,
$U_{(i)}^{(n)} - \frac{i}{n} \longrightarrow 0$ in probability as $n \longrightarrow \infty$.   [5]

4. Consider a random sample $X_1, X_2, \ldots, X_n$ from a continuous distribution with c.d.f. F and suppose we want to test $H_0 : F = F_0$ where F_0 is a fully specified c.d.f. Define the directional and non-directional Kolmogorov-Smirnov test statistics, D_n^+, D_n^- and D_n for testing H_0. Show that, under H_0,
(a) each of these statistics is distribution free;
(b) each of them converges to 0 in probability as $n \longrightarrow \infty$.   [15]

5. Two methods, A and B, were used in a determination of the latent heat of fusion of ice. The investigators wished to check whether the methods differed, and if so, whether method B typically gave a higher reading. The following table gives the change in total heat from ice at $-.72°$C to $0°$C.

Method A	79.97	80.01	79.95	80.02	79.94
Method B	80.05	79.98	80.04	80.03	

Use an appropriate nonparametric method for this investigation.   [10]